1. Calculate the energy of the photon, given the frequency ($\nu = 2\times10^{9}$):

$$E = h\,\nu = 4.136\times10^{-15} \bullet 2\times10^{9} = 8.27\times10^{-6}\ \text{eV}$$
$$= 6.626\times10^{-34} \bullet 2\times10^{9} = 1.325\times10^{-24}\ \text{Joules}$$

2. What is the frequency of a 1 mm wave?

$$\lambda f = c; \quad f = \frac{3\times10^{8}}{1\times10^{-3}} = 3\times10^{11}\ \text{Hz}$$

3. Black - body radiation problem:

$$T = 20 + 273 = 293\ \text{K}$$
$$\text{Power / area} = \sigma T^4 = 5.67\times10^{-8} \bullet 293^4 = 417.9\ \text{Watts / m}^2$$
$$\text{Power} = 4\pi r^2 \bullet 417.9 = 5.25\ \text{kiloWatts}$$

4. At what wavelength does the radiation peak?

For T = 20 C = 293 K

$$\lambda_{max} = \frac{a}{T} = \frac{2.898\times10^{-3}}{293} = 9.89\times10^{-6}\ \text{m}$$

For T = 1000 C = 1273 K

$$\lambda_{max} = \frac{a}{T} = \frac{2.898\times10^{-3}}{1273} = 2.277\times10^{-6}\ \text{m}$$

5. Calculate the equilibrium temperature of a satellite in free space, at a distance from the sun of 1 astronomical unit (1 AU = 93 million miles=1.5 x 10^8 km, Solar Radius $R_\odot$ = 6.96 x 10^5 km). Take the emissivity, ε, to be one for the sun (a good assumption), and the satellite (less so).

a) Calculate the radiated power: $P = \sigma\ T_{sun}^4 \bullet 4\pi R_{sun}^2$

$$P = 5.67\times10^{-8} \bullet 5800^4 \bullet 4\pi \left(6.96\times10^{8}\right)^2 = 3.91\times10^{26}$$

b) Calculate the amount of power which reaches earth - it drops off as Gauss' law might suggest:

$$P = \sigma T_{sun}^4 \bullet 4\pi R_{sun}^2 \bullet \frac{1}{4\pi R_{\text{Earth Orbit}}^2}.$$

$$P = 5.67\times10^{-8} \bullet 5800^4 \bullet \frac{4\pi \left(6.96\times10^{8}\right)^2}{4\pi \left(1.5\times10^{11}\right)^2} = 1381\ \frac{\text{Watts}}{\text{m}^2}$$

You can check that this gives the known answer: 1370 $\pm$ 4 W/m^2

c) The radiation incident on the satellite must be re-radiated:

$$P_{\text{incident}} = \sigma T_{\text{sun}}^4 \bullet 4\pi R_{sun}^2 \bullet \frac{1}{4\pi R_{\text{Earth Orbit}}^2} \bullet \pi R_{\text{satellite}}^2 = \sigma T_{\text{satellite}}^4 \bullet 4\pi R_{\text{satellite}}^2$$

noting that the radiation is incident on the <u>projected</u> area of the sphere, which is the circular disk cross-section. Solve the last equation for the satellite temperature.

$$T_{\text{sun}}^4 \bullet R_{sun}^2 \bullet \frac{1}{4R_{\text{Earth Orbit}}^2} = T_{\text{satellite}}^4$$

$$T_{\text{satellite}} = T_{\text{Sun}} \bullet \left[\frac{R_{sun}^2}{4R_{\text{Earth Orbit}}^2}\right]^{\frac{1}{4}} = T_{\text{Sun}} \bullet \left[\frac{R_{\text{Sun}}}{2R_{\text{Earth Orbit}}}\right]^{\frac{1}{2}}$$

$$T_{\text{satellite}} = 5800 \bullet \left[\frac{6.96\times10^8}{2 \bullet 1.5\times10^{11}}\right]^{\frac{1}{2}} = 5800 \bullet 0.048 = 279 \text{ K}$$

6. For a Bohr atom, find the energy for the n =3 to n = 2 transition.

$$\Delta E_{\text{electron}} = E_f - E_i = -13.58\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = -13.58\left(\frac{1}{4} - \frac{1}{9}\right) = -1.886 \text{ eV}$$

the photon energy is then the negative of this, or +1.886 eV

$$E_{\text{photon}} = \text{h f} = 4.136\times10^{-15} \bullet f = 1.886 \text{ eV}; \quad f = 4.56\times10^{14}$$

$$\lambda \text{ f} = \text{c}; \quad \lambda = \frac{3\times10^8}{4.56\times10^{14}} = 6.58\times10^{-7} \text{ meters}$$

This is termed the H-alpha (H_α) transition

7. Start by converting to Joules:

$$kT = 1.38\times10^{-23} \bullet 1000 = 1.38\times10^{-20} \text{ J}$$

The average kinetic energy of one atom is then given by:

$$\tfrac{1}{2}mv^2 = \frac{3}{2}kT = 1.5 \bullet 1.38\times10^{-20} \text{ J} = 2.07\times10^{-20}.$$

Note that the factor of 1.5 is really a detail....We take the atom to be hydrogen

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2.07\times10^{-20}}{1.67\times10^{-27}}} = 5.0\times10^3 \ {}^{m}\!/_{s}$$

what would the velocity be for oxygen? (mass = $16 * 1.67\times10^{-27}$)

8. $n(K) = \dfrac{2N\sqrt{K}\, e^{-K/kT}}{\sqrt{\pi}(kT)^{3/2}} = \dfrac{2N}{\sqrt{\pi}}\dfrac{1}{(kT)}\sqrt{\dfrac{K}{kT}}\, e^{-K/kT}$;

having rewritten the equation slightly, it should now be obvious that the dimensions are number per energy - ultimately to get number you would integrate over all energies. Taking $N = 10^6$, $kT = 1$.

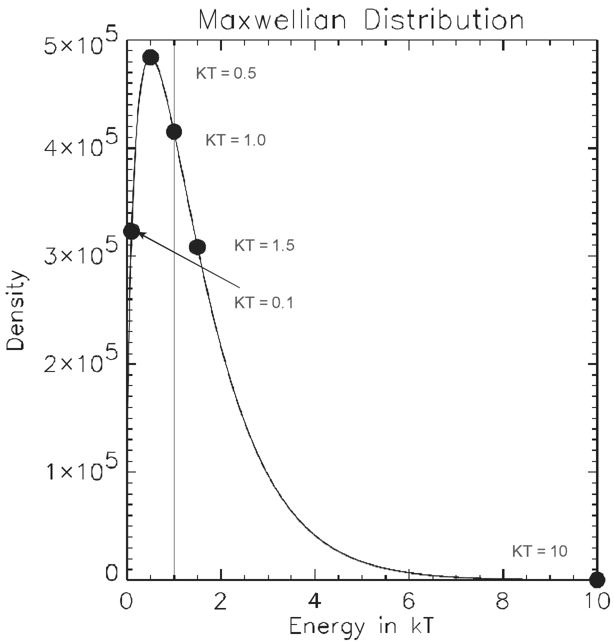
$$n(0.1\ kT) = \frac{2N}{\sqrt{\pi}}\frac{1}{(kT)}\sqrt{\frac{K}{kT}}\, e^{-K/kT} = \frac{2\bullet 10^6}{\sqrt{\pi}}\frac{1}{(1)}\sqrt{\frac{K}{kT}}\, e^{-K/kT}$$

$$= \frac{2\bullet 10^6}{\sqrt{\pi}}\frac{1}{(1)}\sqrt{0.1}\, e^{-0.1} = \ = 1.13\times 10^6 \bullet \sqrt{0.1}\, e^{-0.1} = \ = 1.13\times 10^6 \bullet 0.286 = 3.23\times 10^5$$

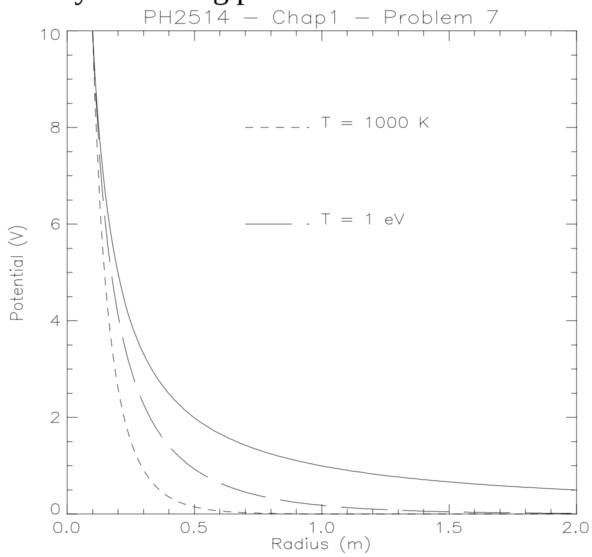
$$n(0.5\ kT) = \frac{2\bullet 10^6}{\sqrt{\pi}}\frac{1}{(1)}\sqrt{0.5}\, e^{-0.5} = 1.13\times 10^6 \bullet 0.429 = 4.84\times 10^5$$

$$n(1.5\ kT) = \frac{2\bullet 10^6}{\sqrt{\pi}}\frac{1}{(1)}\sqrt{1.5}\, e^{-1.5} = 1.13\times 10^6 \bullet 0.273 = 3.08\times 10^5$$

$$n(10\ kT) = \frac{2\bullet 10^6}{\sqrt{\pi}}\frac{1}{(1)}\sqrt{10}\, e^{-10} = 1.13\times 10^6 \bullet 1.44\times 10^{-4} = 1.62\times 10^2$$



9. Debye shielding problem...



The general formula is:

$$\lambda_D\ (\text{meters}) = \left(\frac{\varepsilon_0\ kT}{e^2\ n_e}\right)^{1/2}$$

but it is easier to use the numerical forms:

(A) n = 1.0×10^{11}, T = 1000 K;

$$\lambda_D = 69\left(\frac{T}{n_e}\right)^{\frac{1}{2}} = 69\left(\frac{1000}{1.0\times10^{11}}\right)^{\frac{1}{2}}$$

$$= 69\bullet10^{-4}\ \text{m}=0.69\ \text{cm};$$

and,

(B) n = 1.0×10^{6}, T = 500 eV

$$\lambda_D = 7430\left(\frac{T}{n}\right)^{\frac{1}{2}} = 7430\left(\frac{500}{1.0\times10^{6}}\right)^{\frac{1}{2}} = 166\ \text{m}$$

now just plot up $\Phi = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}\ e^{-(r-r_0)/\lambda_D}$

Note that the solid line in the plot is for no Debye sheath (just a coulomb potential)

11. B = 100 nano-Tesla,

$$B = 100 \times 10^{-9}\ T,\ \ f = \frac{qB}{2\pi m} = \frac{1.6 \times 10^{-19} \bullet 100 \times 10^{-9}}{2\pi \bullet 1.67 \times 10^{-27}} = 1.525\ \text{Hz}$$

12. Use B= 3.1×10^{-5} Tesla is the canonical value

$$f=\frac{qB}{2\pi\ m} = \frac{1.6 \times 10^{-19} \bullet 3.1 \times 10^{-5}}{2\pi \bullet 9.1 \times 10^{-31}} = 0.876 \times 10^{6}\text{Hz} \approx 1\ \text{MHz}$$

13. Drift velocity (E cross B). Note that the drift velocity is independent of energy - the value of 5 eV is extraneous information.

$$E=\frac{\mathbf{E}\times\mathbf{B}}{B^2} = \frac{E}{B} = \frac{10^{-3}}{200\times10^{-9}} = 5000\ \tfrac{m}{s} = 5\ \text{km/s}$$

14 Plasma pressure...

$$p = \sum n_i kT_i = 2\left(2.0\times10^{10}\bullet0.5\right) = 2.0\times10^{10}\ \text{eV}/\text{m}^3$$

$$= 1.6\times10^{-19}\bullet2.0\times10^{10} = 3.2\times10^{-9}\ \text{J}/\text{m}^3$$

(note that the extra "2" above comes from having **both** ions and electrons.

The magnetic energy density (presssure) is given by:

$$p = \frac{B^2}{2\mu_\circ} = \frac{\left(3\times10^{-6}\right)^2}{2\bullet4\pi\times10^{-7}} = 3.58\times10^{-6}\ J/m^3$$

The ratio is $\beta = \dfrac{\sum n_i kT_i}{B^2/2\mu_\circ} = \dfrac{3.2\times10^{-9}}{3.58\times10^{-6}} = 8.94\times10^{-4}$, so the magnetic field dominates